International Journal of Modern Physics A

$b\bar{b}$ DESCRIPTION WITH A SCREENED POTENTIAL

P. González
Dpto. de Física Teórica and IFIC, Universidad de Valencia - CSIC
E-46100 Burjassot, Valencia, Spain

A. Valcarce, J. Vijande
Grupo de Física Nuclear, Universidad de Salamanca
E-37008 Salamanca, Spain

H. Garcilazo
Escuela Superior de Física y Matemáticas, Instituto Politécnico Nacional, Edificio 9
07738 México D.F., Mexico

Received (Day Month Year)
Revised (Day Month Year)

Keywords: Bottomonium; screened potentials

There has been a significant progress in the last decade in deriving precisely, from lattice calculations, the static interaction potential between two heavy quarks. In the so-called quenched approximation (only valence, no sea quarks to start with) a funnel potential containing a linear plus a color-coulomb term is well established [1]. Spin and velocity dependent corrections to this form have been obtained as well [2]. When sea quarks are incorporated (unquenched approximation) the long-distance behavior may change dramatically. As a matter of fact it has been shown in QCD at finite temperature [3] and also in SU(2) Yang-Mills theory [4] that the potential saturates, i.e., it gets a constant value from a certain distance. Physically the saturation of the potential is related to screening: light $q\overline{q}$ pairs are created out of the vacuum between the two heavy-quark sources giving rise to a screening of their color charges. The effect coming out at short distances is that the running of the QCD coupling slows down with the distance. This translates in an effective coulomb strength bigger than in the quenched case. At long distances (bigger than the saturation distance) string breaking may take place: it becomes energetically favorable the recombination of the light q and $\overline{q}$ of the pair with the two heavy sources to form two static-light mesons.

Although saturation has not been proven in QCD [1] one can wonder whether there is any direct indication of it from real QCD and/or phenomenology. We shall

argue that this question has a positive answer. Then we shall construct a potential model incorporating screening for the description of hadrons. Its application to the non-relativistic bottomonium system will allow us to study the phenomenological effect of screening and to predict a lower bound for the saturation distance.

From real QCD the Schwinger-Dyson equation allows to extract the running coupling, α_s, behavior in terms of an effective gluon mass, $M_g(Q^2)$ [5]. This behavior is connected to confinement. Actually when this coupling, particularized for $M_g(Q^2 \to 0) \sim \Lambda_{QCD}$, is implemented into the one-gluon-exchange diagram a confining linear potential comes out. More generally a certain connection can be unveiled by assuming a Yukawa parametrization of the confining potential as proposed long time ago from lattice simulations [6]:

$$\overline{V}_{conf}(r) = \overline{\sigma} r \left(\frac{1 - e^{-\mu r}}{\mu r} \right) = \frac{\overline{\sigma}}{\mu} - \overline{\sigma} r \frac{e^{-\mu r}}{\mu r} \, . \tag{1}$$

By realizing that the linear behavior is recovered for μ, the inverse of the saturation distance, going to 0, the following connection has been proposed [7]:

$$\mu(Q^2) = \Lambda_{QCD} - M_g(Q^2) \, . \tag{2}$$

Then plausible values for the saturation distance, μ^{-1}, for heavy meson scales, go from $\mu^{-1}(Q^2 = (5 \text{ GeV})^2) = 1.25$ fm to $\mu^{-1}(Q^2 = (1 \text{ GeV})^2) = 2.15$ fm.

From phenomenology the finite number of experimentally detected hadronic resonances could well be the reflection of the saturation property of the potential. Indeed from a calculation of the non-strange baryon spectra with a Yukawa-screened interaction a perfect one-to-one correspondence between quark model bound states and experimental resonances can be obtained [8]. For heavy mesons the same parametrization allows for an adequate description of $b\bar{b}$ and to a lesser extent of $c\bar{c}$. However this smooth parametrization does not seem to correspond to more recent lattice indications of a rather abrupt saturation transition [9]. To check the effect of such a fast deconfinement we shall analyze its consequences in the simplest non-relativistic system, bottomonium, by using the potential model:

$$V(r) = \begin{cases} \sigma r - e/r & r < r_{br} \\ \sigma r_{br} - e/r_{br} & r \geq r_{br} \end{cases} \, , \tag{3}$$

where σ is a confining parameter whose value can be guessed from the value of the force at intermediate distances [10] ($\sigma \sim 800 - 900$ MeV·fm^{-1}), e is the unquenched Coulomb strength ($e = 106$ MeV·fm ($\alpha_s = 0.4$)), and r_{br} is the breaking or saturation distance for which we expect 1 fm $< r_{br} <$ 2 fm at the $(Q^2)_{b\bar{b}}$ scale. In order to be more precise we fit the $b\bar{b}$ spectrum. By choosing the b quark mass altogether with σ so as to reproduce the ground state mass and spectral energy separations ($m_b = 4820.5$ MeV, $\sigma = 800$ MeV·fm^{-1}), r_{br} is fitted by assuming that the energy breaking or saturation threshold, E_{br}, is just above the highest energy resonance known, say $E_{br} \sim 11050$ MeV. In this manner a lower bound value for r_{br} ($r_{br} = 1.76$ fm) is in fact obtained. The results for the complete spectrum (up to d states) are shown in Table 1 where the calculated masses are assigned to spin-triplet

states. Since differences in masses with data are at most of 30 MeV an unambiguous identification of states is possible though for the ns states with $n \geq 4$ some room for a slight mixing with d states is left.

Table 1. $b\bar{b}$ spin-triplet bound state masses and properties.

	Mass (MeV)	Exp.	$\langle v^2/c^2 \rangle$	$\langle r^2 \rangle^{1/2}$ (fm)
$1s$	9460	9460.30 ± 0.26	0.10	0.21
$2s$	10022	10023.26 ± 0.31	0.08	0.51
$1d$	10171	10162.2 ± 1.6	0.02	0.56
$3s$	10346	10355.2 ± 0.5	0.09	0.76
$2d$	10444		0.04	0.80
$4s$	10605	10580.0 ± 3.5	0.10	0.97
$3d$	10679		0.06	1.00
$5s$	10829	10865 ± 8	0.11	1.12
$4d$	10889		0.07	1.23
$6s$	11030	11019 ± 8	0.11	1.43
$1p$	9935	9900.1 ± 0.5	0.03	0.41
$2p$	10267	10260.0 ± 0.5	0.05	0.67
$3p$	10531		0.07	0.89
$4p$	10761		0.08	1.09
$5p$	10958		0.08	1.43

This is confirmed by the calculation of leptonic widths, Table 2 (for details of the calculation see Ref. 7), showing a good agreement with experiment (see the fourth column where the effect of radiative and relativistic corrections is partially eliminated for ns states by taking their ratio to the $1s$ case).

Table 2. Leptonic widths $\Gamma_{e^+e^-}$ for $b\bar{b}$ in keV.

	$\Gamma^{(0)}_{e^+e^-}\left(1-\frac{16\alpha_s}{3\pi}\right)$	$(\Gamma_{e^+e^-})_{\mathrm{exp}}$	$\Gamma^{(0)}_{e^+e^-}/\Gamma^{(0)}_{e^+e^-}(1s)$	$[\Gamma_{e^+e^-}/\Gamma_{e^+e^-}(1s)]_{\mathrm{exp}}$
$1s$	1.01	1.32 ± 0.05	1	1
$2s$	0.35	0.520 ± 0.032	0.35	0.41 ± 0.07
$1d$	2.2×10^{-4}		2.2×10^{-4}	
$3s$	0.25	seen	0.25	seen
$2d$	2.6×10^{-4}		2.6×10^{-4}	
$4s$	0.22	0.248 ± 0.031	0.22	0.19 ± 0.03
$3d$	5.7×10^{-4}		5.7×10^{-4}	
$5s$	0.18	0.31 ± 0.07	0.18	0.24 ± 0.06
$4d$	6.7×10^{-4}		6.7×10^{-4}	
$6s$	0.14	0.130 ± 0.030	0.14	0.10 ± 0.03

Electromagnetic E1 transitions have been also evaluated, Table 3, in excellent agreement with data.

Finally the spin singlet-triplet splitting has been calculated perturbatively. Our result $M\left[\Upsilon(1s)\right] - M\left[\eta_b(1s)\right] \simeq 185.7$ MeV agrees with the experimental value

Table 3. E1 decay widths for $b\bar{b}$ in keV.

Transition	Γ_{E1}	$\Gamma_{\rm exp}$
$\Upsilon(2s) \to \gamma\chi_{b_0}(1P)$	1.62	1.7 ± 0.5
$\Upsilon(2s) \to \gamma\chi_{b_1}(1P)$	2.55	3.0 ± 0.8
$\Upsilon(2s) \to \gamma\chi_{b_2}(1P)$	2.51	3.1 ± 0.8
$\Upsilon(3s) \to \gamma\chi_{b_0}(2P)$	1.77	1.4 ± 0.4
$\Upsilon(3s) \to \gamma\chi_{b_1}(2P)$	2.88	3.0 ± 0.6
$\Upsilon(3s) \to \gamma\chi_{b_2}(2P)$	3.14	3.0 ± 0.6

$M\,[\Upsilon(1s)] - M\,[\eta_b(1s)] = 160 \pm 40$ MeV within the experimental errors.

We can then conclude that our naive non-relativistic quark model provides with an excellent description of bottomonium, much better than the corresponding to a non-screened funnel potential and at the level of the best descriptions obtained with much more ellaborated models. These results seem to support the view that a rather abrupt saturation may take place in real QCD. Moreover we predict that at the momentum scale corresponding to $b\bar{b}$ the saturation distance should be bigger than 1.75 fm. From this large value one does not expect saturation can be easily seen in current lattice simulations with the usual Wilson loops techniques. Meanwhile it could be worthwhile to examine at the phenomenological level the effects of an abrupt saturation in other meson and baryon systems.

Acknowledgements

This work was partially funded by Dirección General de Investigación Científica y Técnica (DGICYT) under the Contract No. BFM2001-3563, by Junta de Castilla y León under the Contract No. SA-104/04, by EC-RTN, Network ESOP, contract HPRN-CT-2000-00130, by Oficina de Ciencia y Tecnología de la Comunidad Valenciana, Grupos03/094, and by COFAA-IPN (Mexico).

References

1. G. S. Bali, *Phys. Rep.* **343**, 1 (2001).
2. G. S. Bali, K. Schilling, and A. Wachter, *Phys. Rev.* **D56**, 2566 (1997).
3. C. DeTar, O. Kaczmarek, F. Karsch, and E. Laermann, *Phys. Rev.* **D59**, 031501 (1999).
4. P. de Forcrand and O. Philipsen, *Phys. Lett.* **B475**, 280 (2000).
5. A. C. Aguilar, A. Mihara, and A. A. Natale, *Phys. Rev.* **D65**, 054011 (2002).
6. K. D. Born, E. Laermann, N. Pirch, T. F. Walsh, and P. M. Zerwas, *Phys. Rev.* **D40**, 1653 (1980).
7. P. González, A. Valcarce, H. Garcilazo, and J. Vijande, *Phys. Rev.* **D68**, 034007 (2003).
8. J. Vijande, P. González, H. Garcilazo, and A. Valcarce, *Phys. Rev.* **D69**, 074019 (2004).
9. G. S. Bali *et al.*, *Phys. Rev.* **D62**, 2566 (1997).
10. R. Sommer, *Nucl. Phys.* **B411**, 839 (1994).